# Analysis of Single Queue – Single Server and Single Queue - Multi Server Systems using Simulation: Case Studies of ECO and First Banks

Adamu Muhammad

**Abstract:** This research study is analysis of single queue – single server and single queue - multi server systems using simulation in Banks in Zaria and Minna which are cities in Nigeria. The data collected was analyzed and simulation was performed to reveal the problems associated with the case study Banks. The result revealed that the Banks can reduce customer waiting time as a tradeoff or an opportunity cSSost. Economic analysis of these costs will help the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service.

**Keywords:** Performance measure, Arrival rate, Service rate, waiting line, Cost of waiting or unhappiness cost, Facility cost, Spreadsheet analysis, Simulation (Winqsb), Java program, Queuing theory, Poisson and exponential.

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## 1 Introduction

Queue or waiting in lines is something everyone does and waiting lines are encountered everywhere. When queue management works, your contact center is a healthy organization that represents your company in a positive light; but when queue management fails, your contact center is infected with the "contact center disease" – hold time. When implementing an intelligent Queue Management Strategy, contracting the "contact center disease" or hold time will be avoided, which will increase customer satisfaction and reduce your contact center costs [1].
Extended hold time leads to customer dissatisfaction and defection (churn), Low service levels and high abandonment, decreased contact center efficiency, increased telecom expenses, employee dissatisfaction and turnover [1].

Queuing situations is the idea of uncertainty, for example, inter-arrival times and service times. This means that probability and statistics are needed to analyze queuing situations. [2]. A long wait on hold might turn an otherwise satisfied customer into an angry non-customer [3].


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Three main objectives of the bank simulation study are: (1) Collecting data needed for building accurate model that is useful for taking improvement decisions. (2) Using the model to analyze the behaviors of the existing system and predict its performance at various levels of input parameters (i.e., the arrival rate and service rate). (3) Using the model to conduct "What-if" analysis and enhance performance [4].

The following three solution methodologies can be used to carry out performance measure in queuing phenomenon:
(i) Analytical Results (formula) which give quick answers and insight. Analytic results or analysis is an examination of data and facts to uncover and understand cause-effect relationships, thus providing basis for problem solving and decision making.

(ii) Explicit Simulation which is accurate and realistic models and it has broad applicability. Explicit simulation is the use of hybrid computer for the purpose of simulation.

(iii) Hybrid Simulation which is intermediate solution approach but it combines advantages and disadvantages of analysis and simulation.
Measurement of system performance can be used to describe and analyze system and as well compare alternatives.

The following four analysis methods should be used:
i. After the fact analysis: let the system run some n number times, collect the "real" data and analyze – problems?
ii. Predict some simple trends /projections based on experience – problems?
iii. Develop analytical model based on queuing theory – problems?
iv. Run simulation (not real systems) and collect data to

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analyze –problems? [5]

### 1.1 Cost involved in Queuing System

1. FACILITY COST - cost of (acquiring) services facilities, Construction (capital investment) expressed by interest and amortization , Cost of operation: labor, energy & materials Cost of maintenance & repair , Other Costs such as insurance, taxes, rental of space

2. WAITING COST - may include ill-will due to poor service, opportunity loss of customers who get impatient and leave or a possible loss of repeat business due to dissatisfaction [13].

## 2 Objectives of the Study

To analyze single queue – single server and single queue - multi server systems using Spreadsheet and simulation software (WinQSB), and Java program to measure performance and compare the opportunity cost or the cost of business decision made, which will help to reduce the waiting time of customers in the Banks and total cost related to Bank service facility.

Queuing model is used to overcome the congestion of traffic. This traffic can be of any form and this model is mainly used in a situation where customers are involved, because of this, when the queuing model is being coupled with simulation, it is very much conducive to get solution to solve the problem related to customers [14]. Multi-channel, single phase are found in many Banks today,

## 3The models used:

3.1 The M/M/1 Model

The first model adopted in this work is the easiest waiting line model, which involves a single-server, single-line, single-phase system.

The following Conditions were used for the queuing system at the two banks, which is in accordance with the queuing theory.

1. Arrivals follow a Poisson probability distribution at an average rate of $\lambda$ customers per unit of time.
2. Service times are distributed exponentially, with an average of $\mu$ customers per unit of time. That is, service time varies from one customer to the next and is independent of one another but their average rate is known.
3. The queue discipline is First-Come, First-Served (FCFS) basis by any of the Teller. There is no priority classification for any arrival or service.
4. There is no limit to the number of the queue (that is, arrivals come from an infinite or large population).
5. The service providers were working at their full capacity.
6. The average arrival rate ($\lambda$) is greater than average service rate ($\mu$).
7. The systems I studied will have $p < 1$, if not the number of customers in the system will grow without bound. p is the utilization factor (also called traffic intensity).
8. Servers here represent only Cahiers or Tellers that pay or receive money from customers but not other Bank personnel.
9. Service rate is independent of line length; that is, service providers do not go faster because the line is longer.
10. The customers are patient (that is, no balking, reneging or jockeying) and came from a population that is infinite.

The above conditions were met, so, we develop or use a series of equation that define the queue's operating characteristics as shown on the next page.

A Poisson arrival rate/exponential service time/single server means M/M/1 queue in terms of the standard notation.


**Fig. 1:** A Single Server model (Conceptual model) - the most elementary of queuing model, M/M/1 queuing model [6].

The corresponding formula for the models follows.

Using the above assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas for a single server model:

$\lambda$ = mean arrival rate of customers (average number of customers arriving per unit of time)
$\mu$ = mean service rate (average number of customers that can be served per unit of time)
The average utilization of the system
$P = \lambda / \mu$ …………........... (1)
The average number of customers in the service system
$L = \lambda / (\mu - \lambda)$ ……………..… (2)
The average number of customers waiting in line
$LQ = PL = \lambda^2 / \mu (\mu - \lambda)$ ….. (3)
The average time spent waiting in the system, including service
$W = 1/(\mu - \lambda)$ …………….. (4)

The average time spent waiting in line
$WQ = PW = \lambda / \mu (\mu - \lambda)$ …. (5)

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The probability that there are more than 5 customers in the system which equals one minus the probability there are 5 or fewer customers in the system is given by:

$$p = 1 - \sum_{n=0}^{5} P_n = 1 - \sum_{n=0}^{5} (1-P)P^n$$

... (6)

[7].

The probability that n customers are in the service system at a given time,

.

Pn= (1-P)Pn ……..…………….. (7) [8].

It should be noted that the service rate must be greater than the arrival rate, that is, μ > λ. If $\mu \geq \lambda$, the waiting line will eventually grow infinitely large. Before using the formulas, check to be sure that μ >λ [7].

3.2 The M/M/S Model

The second model adopted in this work is the (M/M/S) :( ∞ /FCFS) - Multi-server Queuing Model. For this queuing system: All of the conditions listed earlier for the single-channel model apply to Multi-server system as well in addition to the following three:
(1) The service times are distributed exponentially, with an average of μ customers per unit of time and number of Tellers s. If there are n customers in the queuing system at any point in time, then the following two cases may arise:
(i) If n < s, (number of customers in the system is less than the number of Tellers), then there will be no queue. However, (S – n) number of Tellers will not be busy. The combined service rate will then be μn = nμ; n < s.
(ii) If n > s, (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be (n – s). The combined service rate will be μn = nμ; n ≥ s.
(2) In this multiple-channel queuing system, two or more servers or channels are available to handle arriving Bank customers. Customers awaiting service form one single line and then proceed to the first available Teller. The arrival process is Poisson with rate λ. Arrivals will join a single queue and enter the first available service channel.
(3) Each of these Tellers or channels has an independent and identical exponential service time distribution with mean 1/μ. In multi-server system, all Tellers are assumed to perform at the same rate. [9]

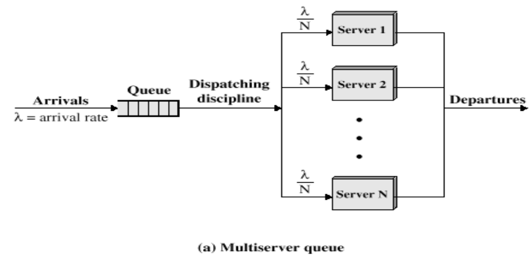


**Fig. 2:** Multi-server model.

Multi-server queue can be modeled as a series of single server queuing systems (M/M/s) in parallel. Each of the single server models serves λ/N users, where λ is the average overall arrival rate and "N" is the number of single server units, figure 3, [11]. The same equation used for single server applies to each sub model, [16].

In the single-line, multi-server, single-phase model, customers form a single line and are served by the first server available. The multi server model assumes all the conditions for single server stated earlier for M/M/1. In addition to that, (i) the M/M/s model assume that there are s identical servers, the service time distribution for each server is exponential, and the mean service time is 1/μ. (ii) For multi server model, the total service rate is greater than the arrival rate, that is, sμ > λ. If sμ $\leq$ λ, the waiting line would eventually grow infinitely large and will give an unstable graph. Using these assumptions, we can describe the operating characteristics for multi server model with the following formulas:
s = the number of servers in the system
The average utilization of the system

p = λ / sμ ……… (8)

The probability that no customers are in the system

$$P = [\sum_{n=0}^{s-1} (\lambda/\mu)^n / n! + (\lambda/\mu)^s / s! + (1/(1-p))]^{-1}$$

…………………….. (9)

The average number of customers waiting in line

$$L_Q = P_0 (\lambda/\mu)^s p / (s! (1-p)^2)$$ ………. (10)

The average time spent waiting in line

$$W_Q = L_Q / \lambda$$ ………….. (11)

The average time spent in the system, including service

$$W = W_Q + 1/\mu$$ ……… (12)

The average number of customers in the service system

L = λW ……………….…,..... (13)

The probability that n customers are in the system at a given time

$$P_n = \begin{cases} \left(\left(\lambda/\mu\right)^n / n!\right)\left(P_0\right) & \text{for } n \le s \\ \left(\lambda/\mu\right)^n / \left(s! \left(s^{n-1}\right) P_0\right) & \text{for } n > s \end{cases}$$

…………………………..… (14).

[12].

## 4 Analysis of First Bank (Single queue-Single Server) and ECO Bank (Single Queue-multi server) banks in Zaria and Minna

### 4.1 Analysis and simulation of First Bank

**Data collected from First Bank**

Data was collected from the First bank from four different Tellers in four days. The data collected from the first Teller is given in Table 1.

TABLE 1
SINGLE TELLER DATA FROM FIRST BANK

| S/NO | ARRIVAL TIME | INTER-ARRIVALTIME | TIME SERVICE STARTS | TIME SERVICE ENDS | SERVICE TIME |
|---|---|---|---|---|---|
| 45 | 9.55 | 1 | 9.57 | 9.58 | 1 |
| 46 | 9.56 | 2 | 9.58 | 10.00 | 2 |
| 47 | 9.58 | 1 | 10.00 | 10.04 | 4 |
| 48 | 9.59 | 0 | 10.04 | 10.05 | 1 |
| 49 | 9.59 | 0 | 10.05 | 10.06 | 1 |
| 50 | 9.59 | 6 | 10.06 | 10.07 | 1 |
| 51 | 10.05 | 1 | 10.07 | 10.09 | 2 |
| 52 | 10.06 | 5 | 10.09 | 10.10 | 1 |
| 53 | 10.11 | 1 | 10.11 | 10.13 | 2 |
| 54 | 10.12 | 0 | 10.13 | 10.14 | 1 |
| 55 | 10.12 | 2 | 10.14 | 10.18 | 4 |
| 56 | 10.14 | 1 | 10.18 | 10.19 | 1 |
| 57 | 10.15 | 2 | 10.19 | 10.21 | 2 |
| 58 | 10.17 | 0 | 10.21 | 10.25 | 4 |
| 59 | 10.17 | 3 | 10.25 | 10.26 | 1 |
| 60 | 10.20 | 1 | 10.26 | 10.28 | 2 |
| 61 | 10.21 | 2 | 10.28 | 10.30 | 2 |
| 62 | 10.23 | 1 | 10.30 | 10.31 | 1 |
| 63 | 10.24 | 1 | 10.31 | 10.32 | 1 |
| 64 | 10.25 | 3 | 10.32 | 10.33 | 1 |
| 65 | 10.28 | 1 | 10.33 | 10.34 | 1 |
| 66 | 10.29 | 2 | 10.34 | 10.37 | 3 |
| 67 | 10.31 | 1 | 10.37 | 10.39 | 2 |
| 68 | 10.32 | 0 | 10.39 | 10.40 | 1 |
| 69 | 10.32 | 2 | 10.40 | 10.42 | 2 |
| 70 | 10.34 | 7 | 10.42 | 10.44 | 2 |
| 71 | 10.41 | 1 | 10.44 | 10.47 | 3 |
| 72 | 10.42 | 1 | 10.47 | 10.49 | 2 |
| 73 | 10.43 | 1 | 10.49 | 10.51 | 2 |
| 74 | 10.43 | | 10.51 | 10.53 | 2 |
| | | | | | |
| SUM | | 49 | | | 55 |
| MEAN | | 1.689655172 | | | 1.833333333 |
| | | (=1.69) | | | (= 1.83) |

The spreadsheet analysis of performance measures or solution for the data in Table 1 is given with the corresponding resultant graphs of the turnaround and response times.

**Queuing Analysis:** Single Server

**Inputs**

| **Time Unit** | **Minutes** | |
|---|---|---|
| Arrival rate (λ) | 1.69 | customers/minute |
| Service rate (μ) | 1.83 | customers/minute |
| **Intermediate Calculations** | | |
| Average time between arrivals | 0.59172 | minute |
| Average service time | 0.54645 | minute |
| **Performance measures** | | |
| | 0.9235 | |

| | | |
|---|---|---|
| Average server utilization (rho) | | |
| Probability the system is empty (P0) | 0.0765 | |
| Average number of customers in the system (L) | 12.0714 | customers |
| Average number of waiting in line (LQ) | 11.1479 | customers |
| Average time a customer spent in the system (W) | 7.14286 | minute |
| Average time a customer spent waiting in line (WQ) | 6.59641 | minute |

Similar data were collected and analyzed for Teller 2, 3, and 4 as was done for Teller one and the summary of the results for Tellers 1, 2, 3 and 4 are shown in Table 1. Turnarounds, performance measures and their corresponding graphs are given below.

TABLE 2
SUMMARY OF THE PERFORMANCE MEASURES FOR FIRST BANK

| Tellers | Λ | μ | P(rho) | P0 | L | Lq | W | Wq | Pn |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.69 | 1.83 | 0.9235 | 0.0765 | 12.0714 | 11.1479 | 7.14289 | 6.59641 | 0.02319 |
| 2 | 2.05 | 2.09 | 0.98086 | 0.01914 | 51.25 | 50.2691 | 25 | 24.5215 | 0.01432 |
| 3 | 0.97 | 1.87 | 0.51872 | 0.48128 | 1.07778 | 0.55906 | 1.1111 | 0.57635 | 2.50E-05 |
| 4 | 1.52 | 2.1 | 0.72381 | 0.27619 | 2.62069 | 1.89688 | 1.72414 | 1.24795 | 0.00217 |

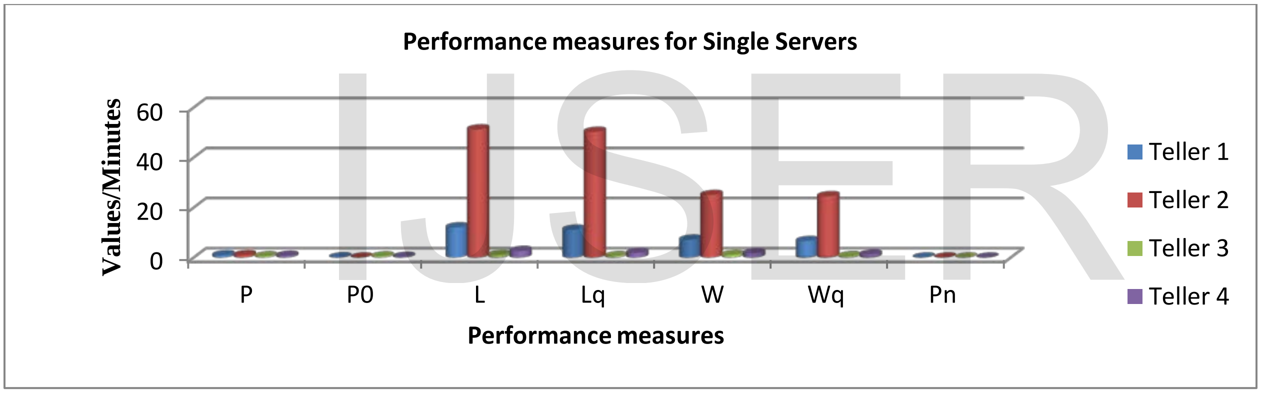


**Fig. 3:** Graph of Performance measures for able 2

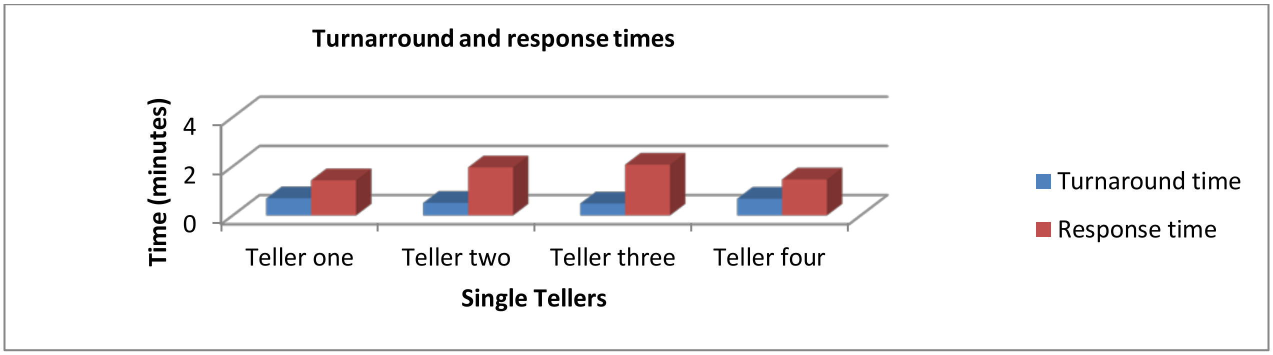


**Fig. 4:** Turnaround and response times for the four First Bank Tellers

TABLE 3
SUMMARY OF COST ANALYSIS FOR THE SINGLE TELLERS OF THE FIRST BANK

| Tellers | Wq (mins) | Unhappiness Cost for 2 hours (N) | Salary Cost (N) | Total Cost (N) |
|---|---|---|---|---|
| **Teller One** | 6.5964 | 4177.72 | 800.00 | 4977.72 |
| **Teller Two** | 24.5215 | 12669.40 | 800.00 | 13469.40 |
| **Teller Three** | 32.8725 | 19175.6 | 800.00 | 19975.6 |
| **Teller Four** | 1.24795 | 603.176 | 800.00 | 1403.18 |

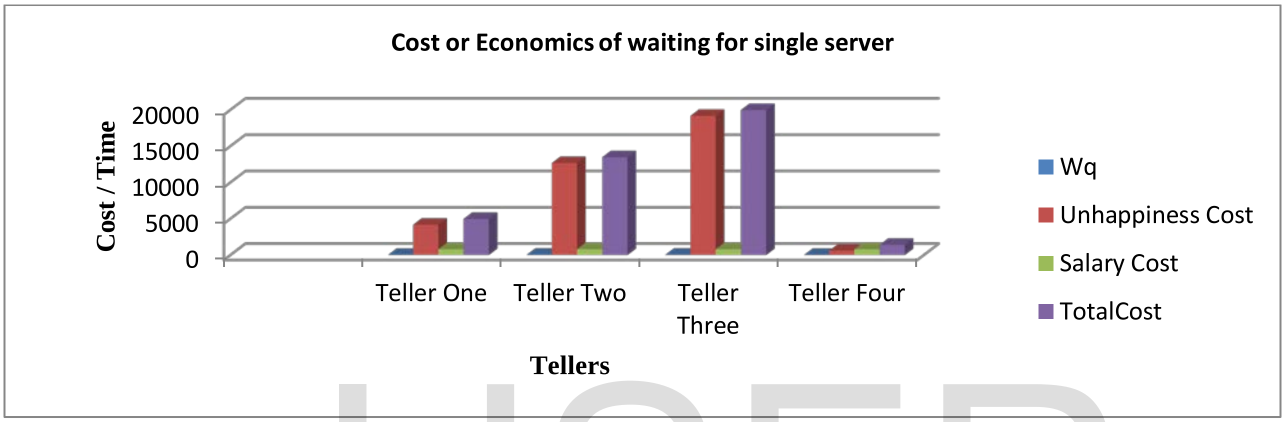


**Fig. 5:** Cost of waiting for First Bank Customers



### 4.2 Analysis and Simulation of ECO Bank

**Data collected from ECO Bank**

Data was collected from the ECO bank from three different Multi-Tellers in three days. The data collected from the first Multi-Teller is given in Table 2. Similar data (not shown) were collected for the two other multi-Tellers. The summary of the Data for the three multi-Tellers is given in Table 2.

TABLE 4
MULTI-TELLER REAL DATA FROM ECO BANK

| S/N O | ARRIVAL TIME | INTER-ARRIVAL TIME | TIME SERVICE STARTS | TIME SERVICE ENDS | SERVICE TIME |
|---|---|---|---|---|---|
| 1 | 8.02 | 1 | 8.16 | 8.19 | 3 |
| 2 | 8.03 | 5 | 8.18 | 8.21 | 3 |
| 3 | 8.08 | 1 | 8.20 | 8.22 | 2 |
| 4 | 8.09 | 3 | 8.22 | 8.24 | 2 |
| 5 | 8.12 | 3 | 8.22 | 8.31 | 9 |
| 6 | 8.15 | 2 | 8.24 | 8.29 | 5 |
| 7 | 8.17 | 3 | 8.29 | 8.30 | 1 |
| 8 | 8.20 | 1 | 8.30 | 8.33 | 3 |
| 9 | 8.21 | 3 | 8.32 | 8.37 | 5 |
| 10 | 8.24 | 2 | 8.33 | 8.35 | 2 |
| 11 | 8.26 | 12 | 8.35 | 8.36 | 1 |
| 12 | 8.38 | 0 | 8.38 | 8.39 | 1 |
| 13 | 8.38 | 2 | 8.38 | 8.44 | 6 |
| 14 | 8.40 | 1 | 8.41 | 8.43 | 2 |
| 15 | 8.41 | 6 | 8.43 | 8.48 | 5 |
| 16 | 8.47 | 0 | 8.47 | 9.00 | 13 |
| 17 | 8.47 | 1 | 8.48 | 9.00 | 12 |
| 18 | 8.48 | 0 | 8.49 | 8.50 | 1 |
| 19 | 8.48 | 0 | 8.51 | 8.52 | 1 |
| 20 | 8.48 | 1 | 8.52 | 8.58 | 6 |
| 21 | 8.49 | 1 | 8.58 | 9.02 | 4 |
| 22 | 8.50 | 4 | 9.01 | 9.10 | 9 |
| 23 | 8.54 | 4 | 9.06 | 9.12 | 6 |
| 24 | 8.58 | 5 | 9.01 | 9.07 | 6 |
| 25 | 9.03 | 5 | 9.08 | 9.15 | 7 |
| 26 | 9.08 | 1 | 9.10 | 9.14 | 4 |
| 27 | 9.09 | 1 | 9.14 | 9.19 | 5 |

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| | | | | | |
|---|---|---|---|---|---|
| 28 | 9.10 | 1 | 9.15 | 9.17 | 2 |
| 29 | 9.11 | 2 | 9.17 | 9.19 | 2 |
| 30 | 9.13 | | 9.19 | 9.27 | 8 |
| | | | | | |
| SUM | | 71 | | | 136 |
| | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| MEAN | | 2.448275862 | | | 4.5333333 |
| | | (ʎ =2.45) | | | (μ=4.53) |

The following are the Spreadsheet solution of the above data, the corresponding graphs for turnaround, performance measure and queuing simulation of the queuing parameters.

**Queuing Analysis:** Multi-server

**Inputs**

| | | |
|---|---|---|
| **Time Unit** | **Minutes** | |
| Arrival rate (λ) | 2.45 | customers/minute |
| Service rate per hour (μ) | 4.53 | customers/minute |
| Number of Servers (s) | 2 | servers |

**Intermediate Calculations**

| | | |
|---|---|---|
| Average time between arrivals | 0.408163265 | minutes |
| Average service time | 0.220750552 | minutes |
| Combined service rate | 9.06 | customer/minute |

**Performance measures**

| | | |
|---|---|---|
| Average server utilization (rho) | 0.270419426 | |
| Probability the system is empty (P0) | 0.574283232 | |
| Average number of customers in the system (L) | 0.583508914 | customers |
| Average number of waiting in line (LQ) | 0.042670061 | customers |
| Average time a customer spent in the system (W) | 0.238166903 | minutes |
| Average time a customer spent waiting in queue (or line) (WQ) | 0.017416352 | minutes |

TABLE 5
SUMMARY OF PERFORMANCE MEASURES FOR THE MULTI-TELERS OF THE ECO BANK

| Multi-Tellers | λ | μ | P (rho) | P0 | Lq | Wq | W | P15 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2.45 | 4.53 | 0.27042 | 0.57428 | 0.04267 | 0.01742 | 0.23817 | 2.18E-08 |
| 2 | 4.48 | 6.70 | 0.33432 | 0.49888 | 0.08414 | 0.01878 | 0.16804 | 2.42E-08 |
| 3 | 3.69 | 5.56 | 0.33183 | 0.50168 | 0.08212 | 0.02225 | 0.20211 | 2.18E-08 |

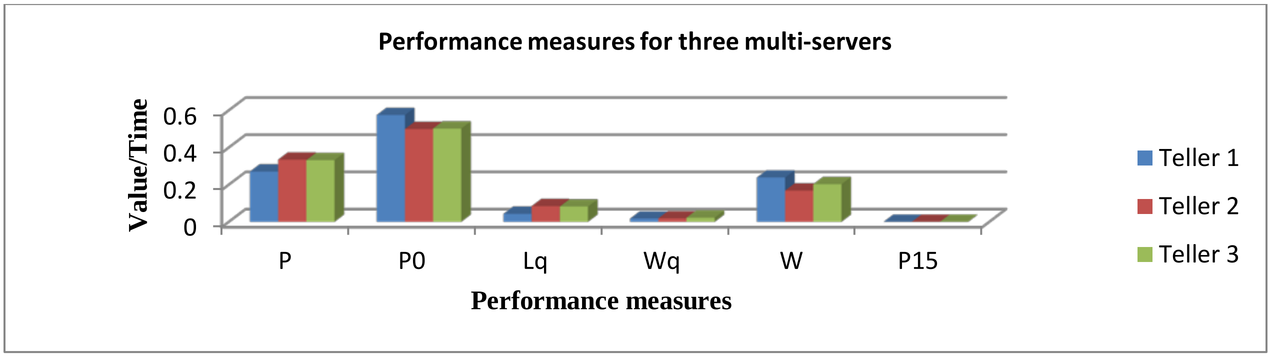


**Fig. 6:** The graph of performance measures for the three ECO Bank multi-tellers

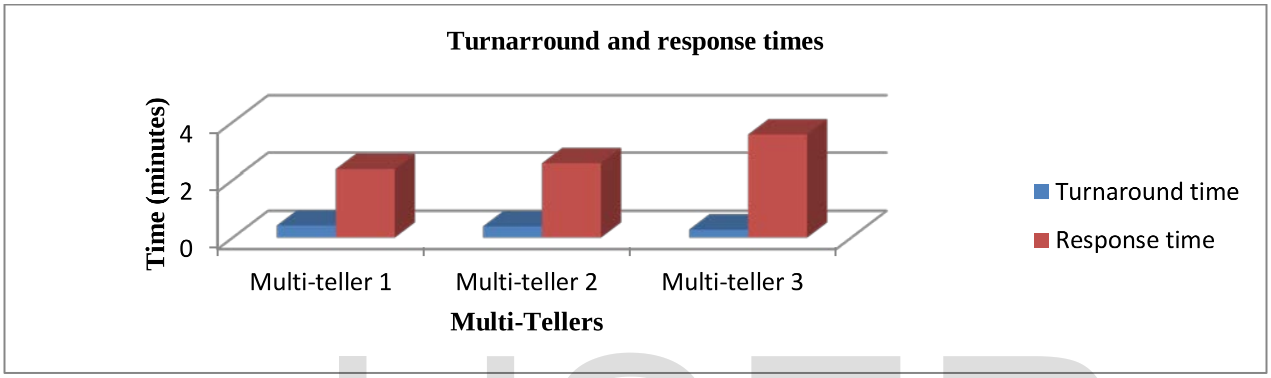


**Fig. 7:** Turnaround and response times for the three Multi-Tellers of the ECO Bank

**ECONOMICS OR COSTS OF WAITING PROBLEM FOR THE M/M/2 MODEL (ECO Bank)**

TABLE 6

COST OF WAITING OR CUSTOMER UNHAPPINESS COST FOR THE ECO BANK FROM THE THREE MULTI-TELLERS

| Tellers | Wq | Unhappiness Cost for 2 hours (N) | Salary Cost (N | Total Cost (N) |
|---|---|---|---|---|
| Multi-Teller One | 0.01742 | 7.25683 | 800 | 807.257 |
| Multi-Teller Two | 0.00291 | 1.64713 | 800 | 801.647 |
| Multi-Teller-Three | 0.02255 | 5.63825 | 800 | 805.638 |

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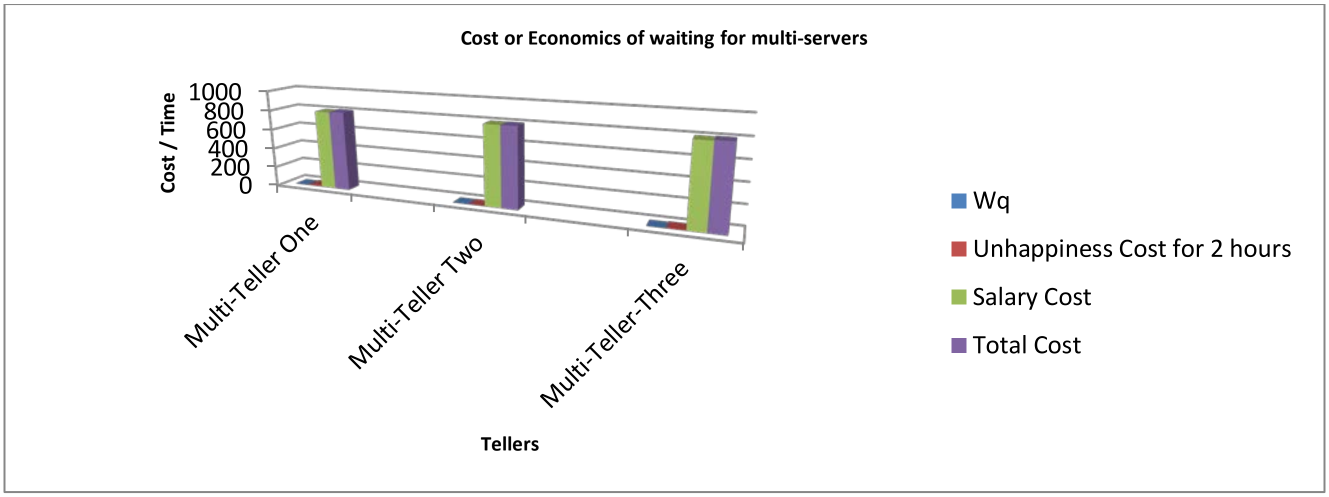
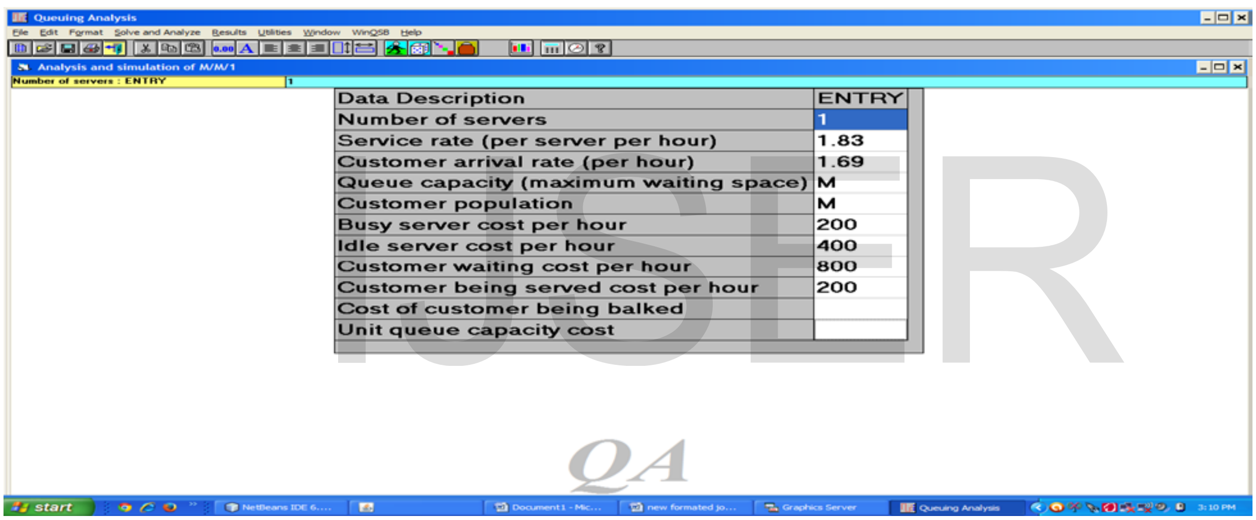



**Fig. 8:** Cost of Waiting for the E Bank Customers from the three Multi-Tellers.




| Data Description | ENTRY |
|---|---|
| Number of servers | 1 |
| Service rate (per server per hour) | 1.83 |
| Customer arrival rate (per hour) | 1.69 |
| Queue capacity (maximum waiting space) | M |
| Customer population | M |
| Busy server cost per hour | 200 |
| Idle server cost per hour | 400 |
| Customer waiting cost per hour | 800 |
| Customer being served cost per hour | 200 |
| Cost of customer being balked | |
| Unit queue capacity cost | |

**Fig. 9:** Data entered for M/M/1 analysis and simulation (Source of WinQSB: WinQSB free trial software) [17, 18]

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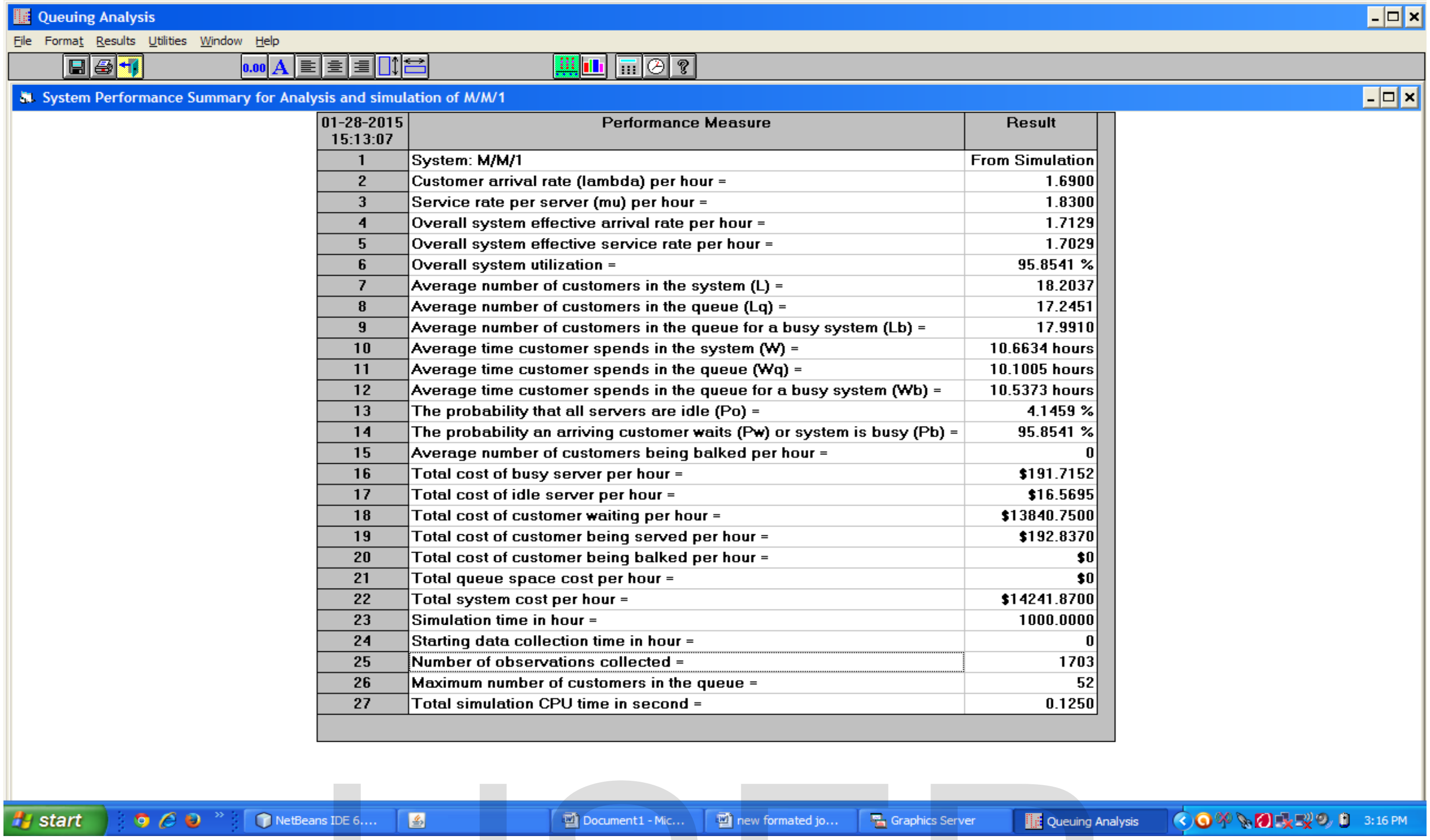
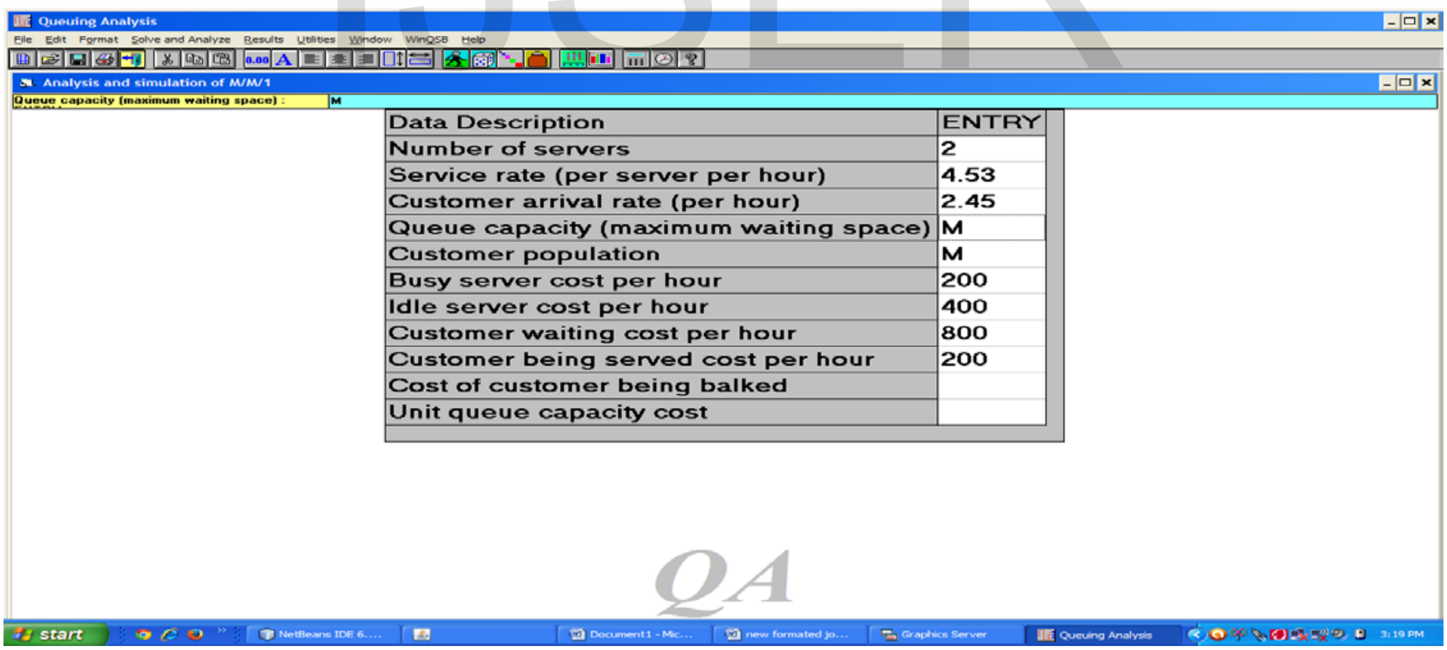



| 01-28-2015 15:13:07 | Performance Measure | Result |
|---|---|---|
| 1 | System: M/M/1 | From Simulation |
| 2 | Customer arrival rate (lambda) per hour = | 1.6900 |
| 3 | Service rate per server (mu) per hour = | 1.8300 |
| 4 | Overall system effective arrival rate per hour = | 1.7129 |
| 5 | Overall system effective service rate per hour = | 1.7029 |
| 6 | Overall system utilization = | 95.8541 % |
| 7 | Average number of customers in the system (L) = | 18.2037 |
| 8 | Average number of customers in the queue (Lq) = | 17.2451 |
| 9 | Average number of customers in the queue for a busy system (Lb) = | 17.9910 |
| 10 | Average time customer spends in the system (W) = | 10.6634 hours |
| 11 | Average time customer spends in the queue (Wq) = | 10.1005 hours |
| 12 | Average time customer spends in the queue for a busy system (Wb) = | 10.5373 hours |
| 13 | The probability that all servers are idle (Po) = | 4.1459 % |
| 14 | The probability an arriving customer waits (Pw) or system is busy (Pb) = | 95.8541 % |
| 15 | Average number of customers being balked per hour = | 0 |
| 16 | Total cost of busy server per hour = | $191.7152 |
| 17 | Total cost of idle server per hour = | $16.5695 |
| 18 | Total cost of customer waiting per hour = | $13840.7500 |
| 19 | Total cost of customer being served per hour = | $192.8370 |
| 20 | Total cost of customer being balked per hour = | $0 |
| 21 | Total queue space cost per hour = | $0 |
| 22 | Total system cost per hour = | $14241.8700 |
| 23 | Simulation time in hour = | 1000.0000 |
| 24 | Starting data collection time in hour = | 0 |
| 25 | Number of observations collected = | 1703 |
| 26 | Maximum number of customers in the queue = | 52 |
| 27 | Total simulation CPU time in second = | 0.1250 |

**Fig. 10:** Performance measures of First and cost from Simulation [17, 18]






| Data Description | ENTRY |
|---|---|
| Number of servers | 2 |
| Service rate (per server per hour) | 4.53 |
| Customer arrival rate (per hour) | 2.45 |
| Queue capacity (maximum waiting space) | M |
| Customer population | M |
| Busy server cost per hour | 200 |
| Idle server cost per hour | 400 |
| Customer waiting cost per hour | 800 |
| Customer being served cost per hour | 200 |
| Cost of customer being balked | |
| Unit queue capacity cost | |

**Fig. 11:** Data entered for M/M/2 analysis and simulation [17, 18]

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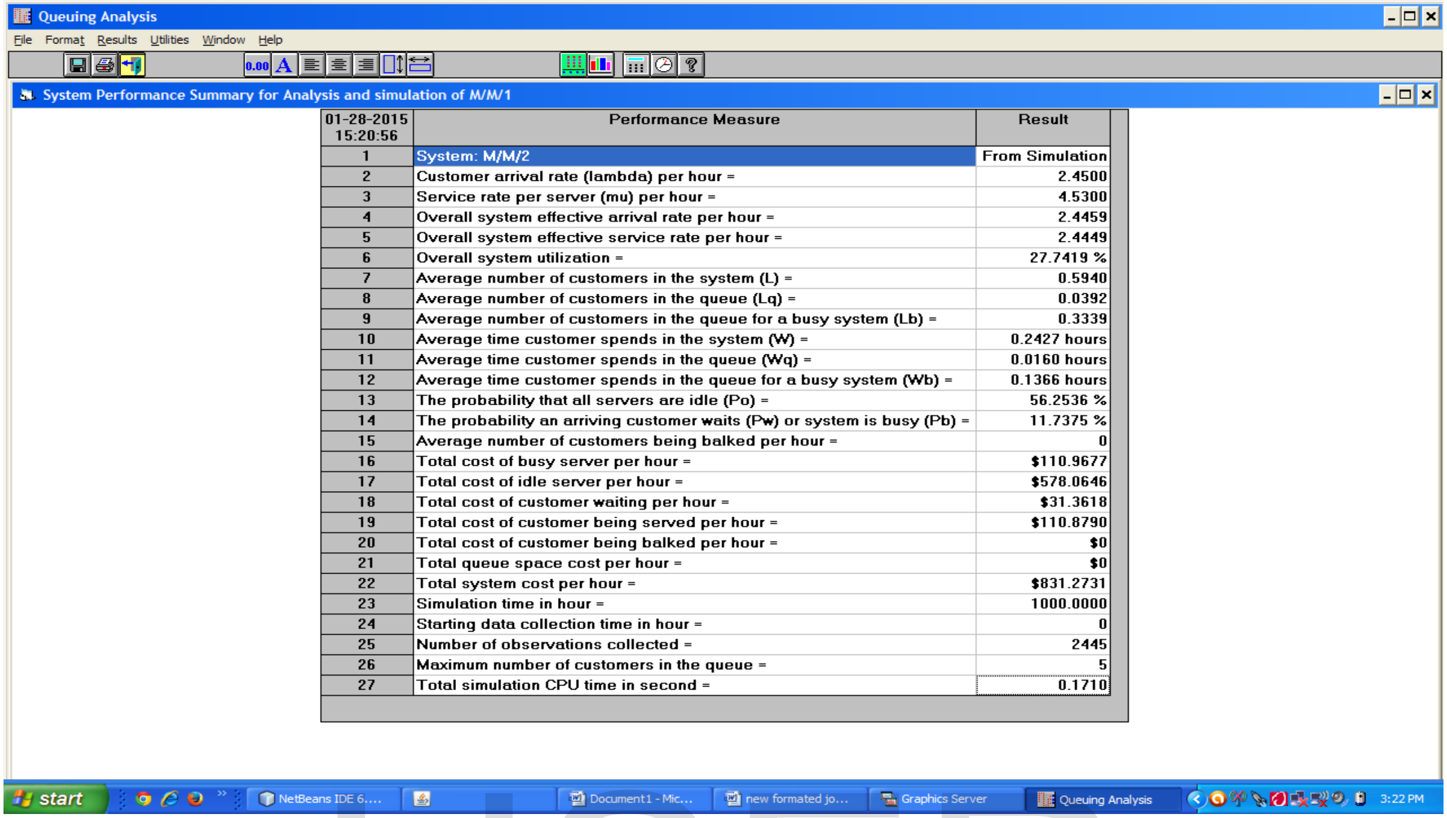
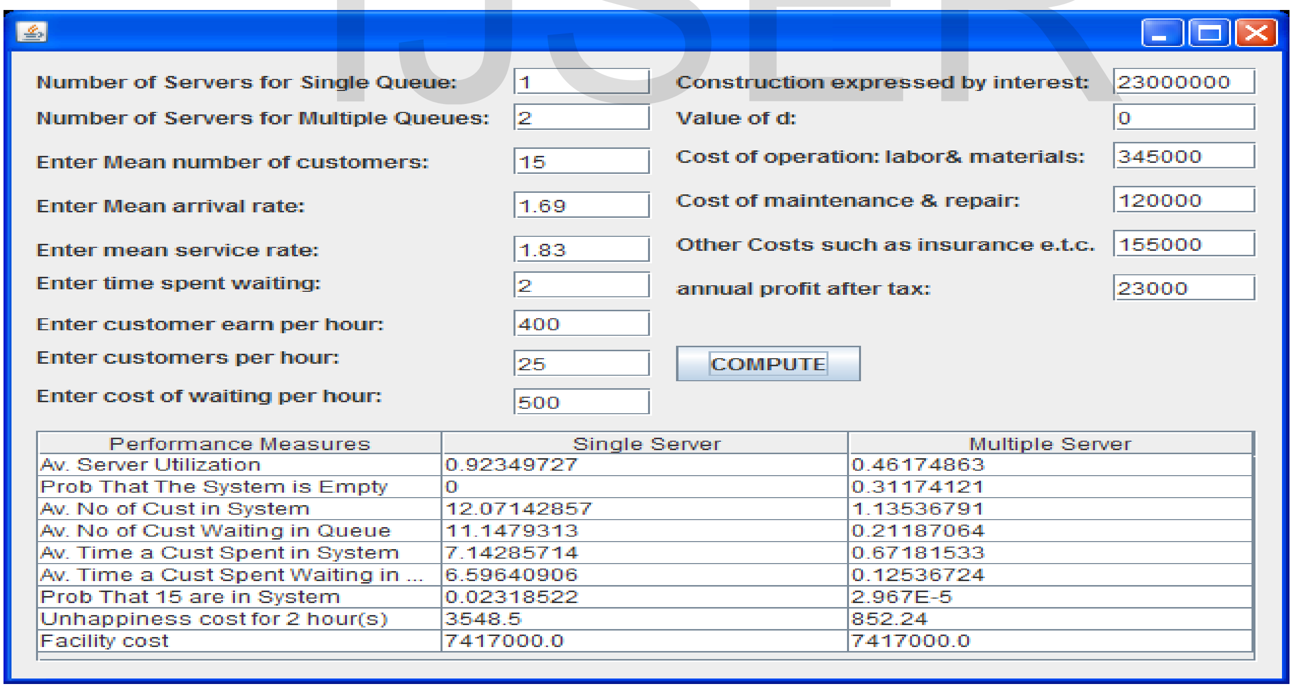



| 01-28-2015 15:20:56 | Performance Measure | Result |
|---|---|---|
| 1 | System: M/M/2 | From Simulation |
| 2 | Customer arrival rate (lambda) per hour = | 2.4500 |
| 3 | Service rate per server (mu) per hour = | 4.5300 |
| 4 | Overall system effective arrival rate per hour = | 2.4459 |
| 5 | Overall system effective service rate per hour = | 2.4449 |
| 6 | Overall system utilization = | 27.7419 % |
| 7 | Average number of customers in the system (L) = | 0.5940 |
| 8 | Average number of customers in the queue (Lq) = | 0.0392 |
| 9 | Average number of customers in the queue for a busy system (Lb) = | 0.3339 |
| 10 | Average time customer spends in the system (W) = | 0.2427 hours |
| 11 | Average time customer spends in the queue (Wq) = | 0.0160 hours |
| 12 | Average time customer spends in the queue for a busy system (Wb) = | 0.1366 hours |
| 13 | The probability that all servers are idle (Po) = | 56.2536 % |
| 14 | The probability an arriving customer waits (Pw) or system is busy (Pb) = | 11.7375 % |
| 15 | Average number of customers being balked per hour = | 0 |
| 16 | Total cost of busy server per hour = | $110.9677 |
| 17 | Total cost of idle server per hour = | $578.0646 |
| 18 | Total cost of customer waiting per hour = | $31.3618 |
| 19 | Total cost of customer being served per hour = | $110.8790 |
| 20 | Total cost of customer being balked per hour = | $0 |
| 21 | Total queue space cost per hour = | $0 |
| 22 | Total system cost per hour = | $831.2731 |
| 23 | Simulation time in hour = | 1000.0000 |
| 24 | Starting data collection time in hour = | 0 |
| 25 | Number of observations collected = | 2445 |
| 26 | Maximum number of customers in the queue = | 5 |
| 27 | Total simulation CPU time in second = | 0.1710 |

**Fig.12:** Data entered for M/M/2 analysis and simulation [17, 18]




| Performance Measures | Single Server | Multiple Server |
|---|---|---|
| Av. Server Utilization | 0.92349727 | 0.46174863 |
| Prob That The System is Empty | 0 | 0.31174121 |
| Av. No of Cust in System | 12.07142857 | 1.13536791 |
| Av. No of Cust Waiting in Queue | 11.1479313 | 0.21187064 |
| Av. Time a Cust Spent in System | 7.14285714 | 0.67181533 |
| Av. Time a Cust Spent Waiting in ... | 6.59640906 | 0.12536724 |
| Prob That 15 are in System | 0.02318522 | 2.967E-5 |
| Unhappiness cost for 2 hour(s) | 3548.5 | 852.24 |
| Facility cost | 7417000.0 | 7417000.0 |

**Fig 13:** One of the Java program outputs for viewing the performance measure of both the case study Banks and the opportunity cost between service provider and customers.

## 5 Discussion of results

The spreadsheet analysis and from simulation (Fig. 9 and Fig. 10) of the First Bank revealed that Tellers 1, 2 and 4 had moderate waiting time while Teller had as high as 24 minutes of wait because of the long queue (Table 2 and Fig. 3) and consequently resulted in higher turnaround and higher customer unhappiness (Fig. 4, Table 3 and Fig.5). This type of Teller 2's higher waiting time is also evident during salary payment time .The Bank Tellers (Servers) were utilized between 50 to 98%. Therefore, this type of higher waiting time and over utilization of the servers can be reduce by bringing more Teller (s) (Servers) into the Bank which will incurred more cost for the service provider. The Bank management should use the Java program developed (Fig. 13) to check trade-offs between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that service before introducing a new Teller in the system (fig. 13). First Bank should turn multi Tellers multi server system to single queue multi-tellers by calling customers in any of their single queue to one of the multi-tellers that his queue has become empty. This will help reduce customer wait time further.

However, the spreadsheet analysis (Table 5) and from simulation (Fig. 11 and Fig.12) of the ECO Bank revealed that customers waited for a very short time for all the three multi-tellers (Table 5 and Fig. 6) compare d to that of the single queue single Tellers and consequently resulted to lower turnarounds (Fig.7) and lower customer unhappiness cost (Table 6 and Fig. 8), but the servers were underutilized (27 to 33%). The ECO Bank should ask their multi-tellers to improve on their turnaround in order to have a further reduction of waiting time and customer unhappiness cost. They do not need to increase server for now, but the management can also use the Java program developed to carry out performance check trade-offs between the increased costs of providing better service and decreased waiting time costs of customers derived from providing that service before introducing a new Teller in the system (Fig. 13). This will help both service provider and customers and as well improve efficiency and save cost.

## 6 Conclusions

The case study Banks and similar service systems should study the discussion of results of this paper and other gray areas uncovered from the analysis and use it to improve their waiting problems and achieve increased efficiency.



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